ELSEVIER
Engineering Analysis with Boundary Elements 26 (2002) 425–433
ENGINEERING ANALYSIS with BOUNDARY ELEMENTS
www.elsevier.com/locate/enganabound

# A scheme for engineer-driven mechanical design improvement

J. Trevelyan[a,*], P. Wang[b], S.K. Walker[c]

[a]School of Engineering, University of Durham, South Road, Durham DH1 3LE, UK
[b]Department of Mechanical Engineering, University of Nottingham, Nottingham, UK
[c]BAE SYSTEMS, Brough, East Yorkshire, UK

Received 27 November 2000; revised 17 March 2001; accepted 9 July 2001

**Abstract**

The role of stress analysis in design is limited by the batch processing of single design geometries when subjected to prescribed load cases. While this approach is reasonable for detailed design, it limits the engineer in early stage design, when the rapid availability of stress information for a wide range of candidate design geometries is perhaps more important than geometric precision. This paper presents structural re-analysis in a boundary element context, providing information on a suitable object-oriented structure and introducing the operations required when processing various types of re-analysis in both (two-dimensional) 2D and more recent developments in (three-dimensional) 3D simulation. Computational performance improvements (over a full analysis) of up to 80% are found in both 2D and 3D, depending on the degree of perturbation to a pre-existing boundary element method model and other factors discussed. Illustrations from an industrial setting show how stress reduction and weight reduction may be achieved as analysis results guide the design in the concept stage.


*Keywords*: Conceptual design; Re-analysis; Design evolution

## 1. Introduction

The use of computational stress analysis tools is well advanced for the detailed design of mechanical components. With a selection of finite element and/or boundary element software packages, an engineer is well placed to determine the stress distribution in, and hence assess the durability of a complex object subject to complex loading. However, commercial systems aimed at solving this type of problem are often ill suited to early-stage design. Here, the emphasis is on obtaining accurate solutions quickly to a range of sketched geometries. A tool which provides this functionality will be useful in determining the validity of a large number of design options at the conceptual design stage.

It is well known that the decisions made during the conceptual design stage are of great importance in determining the extent of the design work, the fundamental durability of the product, the unit cost of the product, and hence the market position of the product. For this reason, it is natural to bring analysis tools forward in the design cycle and allow stress and displacement results to be used in determining product geometries. Clearly, the same argument could be applied to other types of analysis, e.g. electromagnetics, fluid dynamics, etc. but it is linear static stress analysis that is the focus of this paper.

An essential difference between conceptual design and detailed design is the fluid nature of the geometry of the component. While a CAE analyst working in detailed design is likely to be presented with a geometry and a set of loads that are comparatively fixed, the engineer in conceptual design will be considering not only different geometries, but also some very different design topologies. Priorities are also different, in that the emphasis is not so much on geometric precision as on assessing the stress distributions for a range of different topology options.

This paper presents an approach to the conceptual design analysis problem by combining a sketched geometry with a boundary element stress analysis. A key feature of the approach is the structural re-analysis, in which stress and displacement results may be updated efficiently on a design change without the requirement for a complete analysis. Various approaches to re-analysis have been developed, but the method described has the advantage that it can be used efficiently in an evolutionary way, i.e. a model may be modified in one location, and then subsequently in another, and then a third location, etc. The re-analysis allows the engineer to, for example, graphically change a fillet radius

* Corresponding author. Tel.: +44-191-374-7341; fax: +44-191-374-2550.
*E-mail address:* jon.trevelyan@durham.ac.uk (J. Trevelyan).

0955-7997/02/$ - see front matter 
PII: S0955-7997(02)00014-0

and see almost instantaneously the updated stresses on the fillet and surrounding material.

This approach puts design optimisation in the hands of the engineer using tools based on the methodology. Experience will dictate the most effective engineering solution, including not only the stress and displacement constraints, but also those based on the designer's knowledge of materials, manufacturability and adjacent parts. These latter constraints are more difficult to implement in mathematical optimisation schemes, and it seems at this stage that there is a powerful argument for including the engineer in the optimisation. In spite of this, it is clear that the ready availability of re-analysis solutions is a natural requirement of a mathematical optimisation procedure. Sensitivities are easily obtained using re-analysis, and the ideas presented in this paper could equally be applied to optimisation using this approach.

## 2. Boundary element fundamentals

The boundary element method (BEM) [1] has been selected as the tool of choice for this work. This is primarily on account of the ease of re-meshing following an arbitrary geometric change, and then initiating a re-analysis on the new mesh. Readers of this journal will be familiar with the fundamentals of the method, but they are included here for completeness. Readers new to the method should note that it involves formulating the problem as an integral statement in which integrals are performed only over the boundary (surface area) of the component. In this way elements are needed only on the boundary.

The boundary integral equation (BIE) is a statement that describes the reciprocal nature of the relationship between traction and displacement on the boundary of an arbitrary body. Derivations may be found in standard BEM texts. For the case of linear elasticity, in the absence of body forces the BIE may be written

$$c(x)u(x) + \int_{\Gamma_y} t^*(x,y)u(y)\mathrm{d}y = \int_{\Gamma_y} u^*(x,y)t(y)\mathrm{d}y \quad (1)$$

where the primary variables are $u$, representing displacement components, and $t$, which represents surface tractions. The location $x$ is a point at which a fictitious source may be placed, and $y$ represents a general location on the boundary $\Gamma$. The terms $t^*$ and $u^*$ are the so-called fundamental solutions, or free space Green's functions for the problem under consideration. For two-dimensional (2D) elasticity they relate to the traction and displacement at a point $y$ some distance $r$ from position $x$, and are given by

$$t^*_{ij}(x,y) = \frac{-1}{4\pi(1-\nu)r}\frac{\partial r}{\partial n}\left[(1-2\nu)\delta_{ij} + 2\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}\right] + \frac{1-2\nu}{4\pi(1-\nu)r}\left[\frac{\partial r}{\partial x_j}n_i - \frac{\partial r}{\partial x_i}n_j\right] \quad (2)$$

$$u^*_{ij}(x,y) = \frac{1}{8\pi\mu(1-\nu)}\left[(3-4\nu)\ln\left(\frac{1}{r}\right)\delta_{ij} + \frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}\right] \quad (3)$$

where $i$ and $j$ are the directional components, $i$ being the direction of the displacement or traction at the point $y$ and $j$ being the direction of a concentrated point force at $x$. Vector **n** is the unit normal to the surface at $y$, and $n_i$, $n_j$ represent its components in the $i$ and $j$ directions. Terms $x_i$ and $x_j$ are vectors in the $i$ and $j$ directions, respectively. The material properties are contained in $\mu$, the shear modulus, and $\nu$, the Poisson ratio. The term $\delta_{ij}$ is the Kronecker delta. Finally, the multiplier $c(x)$ in Eq. (1) describes the local geometry around point $x$. Most commonly it takes the value of 0.5 on a smooth surface, but it may conveniently be expressed

$$c(x) = \frac{\alpha}{2\pi} \quad (4)$$

where $\alpha$ is the angle subtended by the material at $x$. For three-dimensional (3D) cases it will become necessary to consider $\alpha$ as a solid angle.

In order to maintain brevity in this section of the paper, it will be sufficient to state that similar expressions exist for 3D analysis, and that these may be found in a BEM text.

Having established the analytical basis of the BIE in Eq. (1), it is clear that the integrals are, for general boundaries $\Gamma$, to be performed numerically. This is normally done using a standard Gauss quadrature scheme, though for cases in which the distance $r \to 0$ some coordinate transformation is often used, the most popular being that due to Telles [2]. Numerical integration requires the discretisation of $\Gamma$ into elements. By describing the variation of traction and displacement over each element in terms of a vector of shape functions, $N$, and values of these variables at the nodes of the element, Eq. (1) may be written in its discretised form

$$c(x)u(x) + \sum_{\text{elems}} \int_{\Gamma_{\text{elem}}} t^*(x,y)N^{\mathrm{T}}\,\mathrm{d}y u_{\mathrm{e}} = \sum_{\text{elems}} \int_{\Gamma_{\text{elem}}} u^*(x,y)N^{\mathrm{T}}\,\mathrm{d}y t_{\mathrm{e}}. \quad (5)$$

Vectors $u_{\mathrm{e}}$ and $t_{\mathrm{e}}$ contain the values (as yet unknown) of displacement and traction at the nodes of the element under consideration. By evaluating the integrals numerically, Eq. (5) provides an expression that relates the nodal values of displacement and traction by a set of coefficients. Considering the point $x$ at each node in turn, the expressions thus derived form a set of equations conveniently expressed in matrix form as

$$\mathbf{Hu} = \mathbf{Gt} \quad (6)$$

where **H** and **G** are, respectively, square and rectangular matrices containing the integrals of the $t^*$ and $u^*$ terms, and **u** and **t** are vectors containing the (as yet unknown) nodal displacements and tractions. The $c(x)$ terms are built in to the diagonal coefficients of **H**. Application of a mixed set of boundary conditions, i.e. some displacements

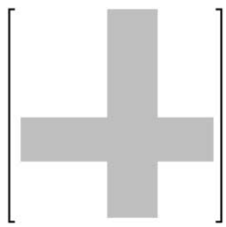

Fig. 1. Modified portions (shown shaded) of the system matrix in a typical BEM re-analysis.

and some tractions, allows Eq. (6) to be consolidated to a familiar form

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{7}$$

where $\mathbf{A}$ is a fully populated, unsymmetric, square matrix containing some columns of $\mathbf{H}$ and some columns of $\mathbf{G}$, $\mathbf{b}$ is a vector derived from $\mathbf{H}$, $\mathbf{G}$ and the boundary condition, and $\mathbf{x}$ remains unknown, being those terms in $\mathbf{u}$ and $\mathbf{t}$ which remain unknown. This set of equations may be solved using a suitable scheme, such as direct Gauss elimination with partial pivoting, or an unsymmetric iterative solver such as GMRES [3] with suitable preconditioning.

Finally, returning to Eq. (5), this may be used again after solution of Eq. (7) so that all terms in $u_e$ and $t_e$ are known. At this stage we may consider any point $x$ wholly within the domain, such that $c(x) = 1$. Evaluating the integral terms in Eq. (5) allows the calculation of $u(x)$, the displacement at the internal point. Similarly, a derivative of Eq. (5) may be used to find stress components. This internal information is essential for accurate contouring of results over planar domains.

## 3. Re-analysis

The foregoing section has shown the fundamentals of obtaining a solution for displacement and traction over a general body under a well constructed set of boundary conditions. Having found such a solution, consider now the problem of updating the analysis results following a small geometric change. This is termed re-analysis, static re-analysis or partial re-analysis.

Re-analysis has been investigated by numerous authors, in particular with reference to the finite element method (FEM). Reviews of early finite element procedures are presented by Arora [4] and, later, by Abu Kassim and Topping [5]. More recently, FEM re-analysis has been presented in an object-oriented framework by Mackie [6], who also takes advantage of substructuring and multi-threading to improve performance. This last work is also evolutionary, in the sense described before, allowing subsequent geometric modifications to be considered with no efficiency penalty. However this, like all FEM implementations, suffers from the difficulty of re-meshing automatically following a potentially substantial geometric change.

Re-analysis can be considered in a number of different ways within a BEM framework. Recall that the system matrix, $\mathbf{A}$, is dense and unsymmetric, precluding the use of many FEM resolution procedures. We denote the updated system, following geometric design change as

$$\mathbf{A}_0\mathbf{x}_0 = \mathbf{b}_0 \tag{8}$$

where $\mathbf{A}_0 = (\mathbf{A} + \Delta\mathbf{A})$, $\mathbf{b}_0 = (\mathbf{b} + \Delta\mathbf{b})$ and we seek the updated solution vector $\mathbf{x}_0 = (\mathbf{x} + \Delta\mathbf{x})$. We consider first the applicability of the various methods for updating matrix inverses. While an efficient procedure for computing the solution to Eq. (7) would not explicitly compute the inverse of $\mathbf{A}$, it is a possibility to be considered if such an inverse can be updated sufficiently rapidly. In other words, we may choose to take the computational penalty of an initial solution via formation of an inverse if there is sufficient performance gain to be achieved in the re-analysis phase. We may then, when we start the re-analysis, have the complete original system in the form

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} \tag{9}$$

and seek the solution to

$$\mathbf{x}_0 = \mathbf{A}_0^{-1}\mathbf{b}_0 \tag{10}$$

leading us to the desirability of a rapid solution to the updated inverse $(\mathbf{A} + \Delta\mathbf{A})^{-1}$, where $\mathbf{A}^{-1}$ is known. A classical approach to this problem is provided by the Sherman–Morrison formula, which may be found in, say, Press [7]. Unfortunately, this is efficient only for problems in which $\mathbf{A}$ and $\mathbf{A}_0$ differ only in a few rows or columns. A BEM re-analysis will, for single zone problems, involve changes to a large number of rows and columns, a typical pattern being shown in Fig. 1.

For this type of modification, update of the matrix inverse by Sherman–Morrison may still be carried out, for example one row at a time, but this approach would incur a similar run time cost to the complete factorisation of the updated matrix by a direct solver.

Castillo et al. [8] presented an alternative formula for updating matrix inverses. However, this approach is also seen to be computationally expensive for matrices modified to the extent shown in Fig. 1. It may be shown that, for the resolution of an $(n \times n)$ system in which $m$ rows and columns have been updated, a total number of floating point operations equal to $2n^2(4m + 1)$ will be required. It will later be demonstrated that this is not competitive with other methods for re-analysis of the type of system considered in this paper.

Kirsch and Toledano [9], and later Kane et al. [10], presented an iterative method for re-analysis of BEM systems. The method may be considered by expanding Eq. (8) after making substitutions of the form $\mathbf{A}_0 = (\mathbf{A} + \Delta\mathbf{A})$, to arrive at

$$\mathbf{A}\,\Delta\mathbf{x} = (\Delta\mathbf{b} - \Delta\mathbf{A}\mathbf{x}) - \Delta\mathbf{A}\,\Delta\mathbf{x}. \tag{11}$$

The term in brackets is fully known at the time of re-analysis,

Table 1
Estimated floating point operations (millions) for solution phase of re-analysis (matrix size $n$)

| | Sherman–Morrison [7] | Castillo [8] | Kane [10] | Leu [11] | Trevelyan and Wang [14] |
|---|---|---|---|---|---|
| $n = 200$ | 48 | 19 | 2.6 | 1.0 | 2.2 |
| $n = 300$ | 162 | 58 | 5.8 | 2.2 | 4.9 |
| $n = 400$ | 384 | 154 | 10.2 | 3.8 | 8.6 |

and may be evaluated. The presence of the vector $\Delta\mathbf{x}$, representing the perturbation to the solution vector, on both sides of the expression leads to an iterative solution. This method may be extended by a scaling of $\mathbf{A}$, and a reformulation of the recurrence formula, which combine to give better convergence properties and thereby allow more substantial geometric changes to be made before the solution diverges.

More recently, Leu [11] presented an iterative scheme for re-analysis of BEM systems following a geometric change, based on a reduction method. Here the updated solution vector is expressed as a linear combination of a reduced set of basis vectors in a manner similar to conjugate gradient solution methods. This has the advantage that only a few basis vectors may be required to achieve the desired accuracy. The method was applied to a set of shape optimisation problems. This is one of a series of reduction methods applied to structural mechanics, and re-analysis, a useful review of which is presented by Noor [12]. These methods have been extended in application to grillages by Kirsch and Moses [13].

The iterative method of Kane and the reduction method of Leu suffer from the fact that they require the inverse, or some triangular factorisation, of the original system matrix $\mathbf{A}$. Although this does not strictly preclude an evolutionary analysis, it requires that any design modifications made after the full first analysis are considered together as a total perturbation to the original geometry. As further geometric modifications are made, this will necessarily give rise to some degradation in performance as a result of larger numbers of iterative cycles, and eventually to a divergent solution. Moreover, after two or three design changes, it may well be that the analysis of the latest geometry is being performed by considering it as a perturbation from a geometry which may now bear little relation to the latest design. This appears undesirable, at least intuitively. The extension of the iterative methods to keep updating the inverse would give rise to the problems already discussed with Sherman–Morrison, and of course would preclude the need for such an iterative scheme in the first place.

Trevelyan and Wang [14] presented a simple re-analysis scheme based on the use of an iterative solver (GMRES). In this approach, the previous matrix $\mathbf{A}$ is overwritten and a full matrix solution performed for each re-analysis. The solution vector from the previous iteration, i.e. vector $\mathbf{x}$, is used as the initial estimate of the perturbed solution vector, $\mathbf{x}_0$. This has the advantage of being evolutionary, since the matrix level description of the problem updates with each modification. No inverse $\mathbf{A}^{-1}$ is required.

In spite of the different evolutionary characteristics of the various methods, it is nevertheless of interest to draw comparisons between the various approaches in terms of computational efficiency for a single re-analysis run. We assume that the methods all incur the same computation in the integration phase, so that the comparison may be made wholly on solution speed. Some enhancements can be made to accelerate the integration phase [15], but these are neglected in this comparison. It is also assumed that the accuracy of the methods is equivalent, and that convergence tolerances on iterative solvers are set appropriately.

Any comparison of computational performance will, of course, be strongly dependent on the number of iterations taken by the various methods to reach a converged solution for the re-analysis problem. Here we can be only approximate. Kane [10] and Leu [11] quote typical examples of 5–10 iterations and 4 iterations, respectively, for small to medium size plane stress cases. Tests using the Concept Analyst software of Trevelyan [12] show the number of GMRES iterations to be consistently around 25 for convergence for similar problems. Based on this information, a rough estimate of the number of floating point operations required for the re-analysis solution may be found, as is shown in Table 1.

These figures should be considered significantly approximate so that only general conclusions may be drawn. It is clear that schemes based around updating matrix inverses are uncompetitive for BEM re-analysis applications. This arises from the large portion of the matrix $\mathbf{A}$ which becomes perturbed. The other methods are broadly similar in terms of solution performance, with the basis vector approach of Leu perhaps showing some advantage. More detailed comparison cannot be made, since the complexities of problem dependence and convergence tolerances become important. Note, however, that the scheme of Kane has been shown to be divergent for higher degrees of geometric perturbation, while those of Leu and Trevelyan are guaranteed to converge given sufficient iterations. Note also that the schemes of Kane and Leu are likely to require an increasing number of iterations with each subsequent re-analysis, while that of Trevelyan will be unaffected.

It should be recalled that Table 1 considers only the solution to the perturbed equations, and not the generation of the perturbed system.

The scheme proposed by the current authors seems to be favourable. It is faster than the other schemes for the first full analysis (not requiring an inverse to be formed explicitly), it is competitive with the other iterative schemes for the first re-analysis, and for subsequent re-analyses it should be seen to be more attractive. Furthermore, the importance of the matrix continually being updated to reflect the latest design geometry cannot be understated.

## 4. Implementation and computational performance

A prototype analysis system entitled Concept Analyst has been produced, in which the re-analysis ideas proposed above have been implemented. The software is developed in Visual C++ for ease of preparing a standard Windows environment. The 2D analysis code has become advanced to the stage at which it is providing high quality solutions very rapidly in an industrial setting. Following the success of the 2D code, a 3D analysis and re-analysis system has also been developed, though currently lacks a graphical front-end. In an academic sense, though, the work that has been done is sufficient to enable conclusions to be drawn about the effective use of re-analysis in three dimensions. Details of the object-oriented implementation in 2D are presented by Trevelyan and Wang [14]. The 3D software follows a similar structure.

The user interface to Concept Analysis has been developed primarily for ease and speed of use. The geometry is built using presentation graphics style commands, in which circles and rectangles may be quickly sketched. General polygons may be defined and deformed interactively. Loads and boundary conditions are applied to geometric edges. Since the meshing and internal point generation are completely automated, the user may proceed to a solution by simply building a sketched geometry and applying loads to geometric entities. This approach is highly suitable to the conceptual design stage, since a sketched geometry is often sufficient to enable the engineer to gain a rapid assessment of the stress distributions.

To illustrate the speed of model creation, and indeed of the analysis, the authors have demonstrated a simple example of a circular hole in a rectangular plate, the plate being subjected to uniaxial tension. The total time taken from *starting to* build the model until a set of stress contours is displayed on the screen is just 12 seconds. The authors are unaware of any more rapid approach that offers the generality of geometry allowed by FEM or BEM numerical analysis. While this speed of modelling is allowed by combining a BEM solution into a sketching system, Concept Analysis allows the geometry to be defined with rigorous care over coordinates as does all commercial FE/BE software.

This speed of response is achieved in a number of ways, including:

- The analysis and graphical display are completely integrated, sharing a common database, precluding the need for time-consuming file i/o operations.
- The geometric definition is intuitive and, at its highest level, characterised in terms of 'shapes', i.e. general planar figures. Shapes may be circular or general polygons, and these are defined using high level commands. This makes it unnecessary, for example, to define a rectangular plate by a set of lengthy point and line operations.
- Material properties are selected from a list of materials. This is considerably faster than requiring the user to specify material properties individually.

Once a set of results contours are displayed, a geometric change may initiate an automatic re-analysis. The modified line segments (in 2D) are identified, and the model remeshed. The re-analysis adopts a suitably intelligent re-meshing algorithm, in which the total number of elements on the modified boundary lines is redistributed between the lines according to conventional meshing rules. The internal points are mapped to new locations depending on the type of design change, though the triangulation between them (for contouring purposes) remains as before. This procedure may involve some overlapping contour triangles, but this is not seen as a particular drawback.

The re-analysis proceeds according to the following schedule:

- A list of changed elements and changed nodes is created. The changed elements include those adjacent to the perturbed geometry, since it will become important to re-compute accurately the matrix coefficients relating to the 'first' and 'last' nodes on the perturbed geometry. System matrix coefficients relating to the changed nodes are initialised to zero.
- Integration is performed considering source point – field element pairs as follows:
  source point at all changed nodes → all elements as field element,
  source point at all nodes → all changed elements as field element.
- Duplicate pairs are of course integrated only once. Note that the diagonal terms in the **H** matrix may still be computed using the row sum method, even though the **H** matrix is not stored. This is because any diagonal term requiring recalculation will lie in a row for which all terms are being recalculated. Diagonal coefficients relating to unperturbed degrees of freedom remain as before, which implies that the row sum in these rows can be expected to be non-zero. However, since the diagonal terms represent the self-influence functions, they are a property of the (undisturbed) local element geometry and are notionally independent of the changes made to remote elements. Standard integration schemes may be used, but it has been found effective to retrieve integrals

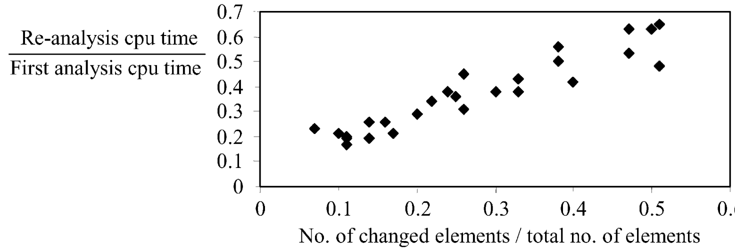


Fig. 2. Re-analysis performance for 2D applications.

from look-up tables as functions of the geometry of the field element relative to the source point [15].

- An iterative solution is initiated in which the initial 'guess' is taken as the solution to the previous geometry. The GMRES solver in Concept Analysis uses a simple diagonal preconditioner.
- Internal point solutions are recalculated as required.

The computational time savings achievable using this method are presented in Fig. 2 (published earlier [14]) and Fig. 3 (new to this work), for 2D and 3D applications, respectively. The results shown are a variety of small and large models, using a variety of types of load (i.e. problems dominated by bending, tension, etc.), and considering a variety of degrees of geometric change.

The time savings found for 2D and 3D re-analysis are of similar order, and show similar trends in Figs. 2 and 3. In both cases, the re-analysis runs in as little as ca. 20% of the full run time if only 10% of the elements are modified, and runs in ca. 60% of the full run time if half the elements are changed.

While the 2D re-analysis performance has been found to be largely independent of the degree of geometric change, this factor does have some influence on re-analysis run-times in 3D problems. Both the re-integration and the resolution phases contribute. The re-integration time is extended for larger geometric modifications because the resulting element distortion requires more integration points for accurate computation. Element distortion in 2D is not a consideration with a competent automeshing algorithm. The fact that the resolution phase benefits from a small geometric change in 3D, though not in 2D, indicates that the **A** matrix may have a better inherent conditioning in 3D elastostatics. While this improved conditioning is not unexpected, it is a useful benefit in the more numerically intensive 3D solution that the equation solver rewards the good first guess provided by the previous solution vector.

The increase in re-analysis times with the number of changed elements is entirely due to the re-integration phase. The resolution time is independent of the number of changed elements. At first sight this appears to be in conflict with the previous paragraph. However, it is found that the resolution times are almost identical when the same geometric modification is achieved by varying the mesh in different ways, using different numbers of modified elements.

Larger problems clearly exhibit better savings in 3D re-analysis than do smaller problems. It is possible that this is related to the addition to the list of changed elements of the set of elements adjacent to the modified portion of the design. While these elements have no change in geometry themselves, it is necessary to integrate over the elements as they share nodes with modified elements. The matrix coefficients relating to these shared nodes must be fully re-computed including the contributions from all elements of which they form part. The set of adjacent elements will form a greater percentage of the total elements in a small problem, and therefore the apparent re-analysis time will increase relative to the full analysis.

Different forms of loading give rise to different re-analysis performance. No discernible pattern is found indicating that bending re-analysis outperforms re-analysis under tension, or that any form of loading consistently outperforms another. This is somewhat unpredictable, though for any single mesh the improvement of one form of loading over another seems to be consistent.

While Figs. 2 and 3 show the 2D and 3D re-analysis to offer broadly similar run time improvements for the re-analysis, it is important to note that care needs to be taken in 3D to minimise the number of changed elements. In 2D the re-meshing of line elements on geometric lines is simple, and we have very little flexibility in this stage of the process. However, in 3D there is a large variety of ways in which a geometric change can be reflected in a new mesh. Many design changes cause changes to boundary surfaces that can be accommodated by re-meshing only a few elements on the surface, e.g. consider a rectangular patch

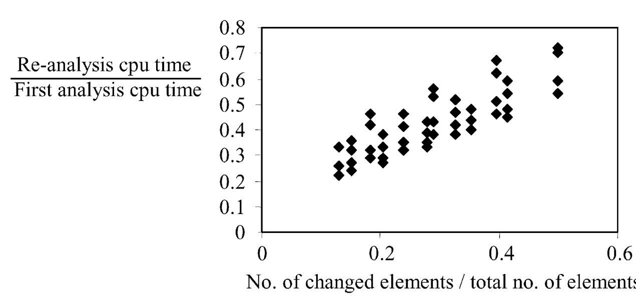


Fig. 3. Re-analysis performance for 3D applications.

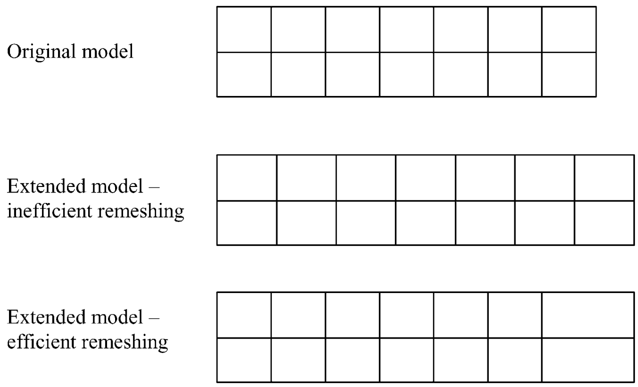


Fig. 4. Re-meshing a rectangular surface extended in its original plane.

that is extended while remaining in plane (Fig. 4). This may be accommodated by extending every element by a uniform scaling, though it will clearly be beneficial to extend only one row of elements. It is re-meshing issues such as this that will become the focus of research in this area.

## 5. Industrial applications

The ability to obtain stress solutions with a very rapid response has far reaching consequences for mechanical design and analysis. The particular set of applications discussed in this section reflects a class of problems frequently encountered in aerostructure engineering. The class of problems discussed is limited to plane stress cases, involving the simulation of a variety of parts subject to stress concentrations. The analysis needs in this case are of two types:

- General design analysis of aircraft parts, which frequently include holes and other geometric features. The features cause stress concentrations whose stress raising effects may be difficult to find through handbooks such as Peterson [16] because of the complex interactions between neighbouring features.

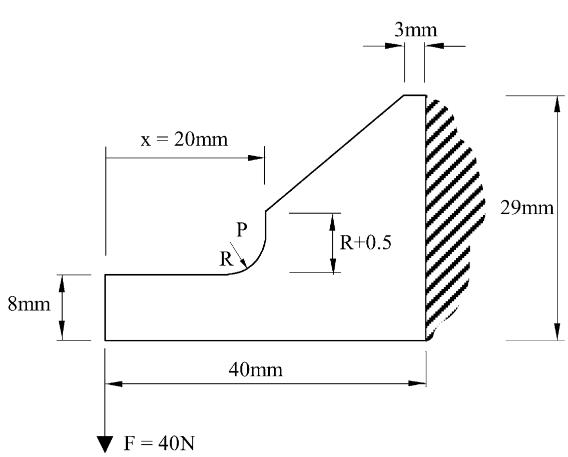


Fig. 5. Integrally machined stiffener.

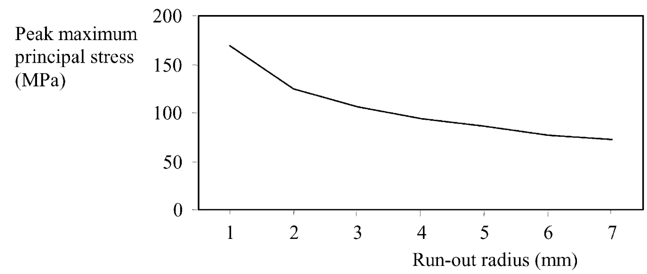


Fig. 6. Reduction in stress concentration factor.

- Concessions, in which a rapid decision is required for the certification for usage of a part which has been manufactured outside the specified tolerance.

Fig. 5 shows an investigation into the effect of increasing the run-out radius, $R$, at the termination of an integrally machined stiffener in an aircraft component. The objective is to reduce the stress concentration factor by at least 10–20% in order to improve the fatigue life of the component. The material is an aluminium alloy and the applied load of 40 N gives rise to a bending moment of 800 N mm at $x = 20$ mm.

The problem was analysed using a standard BEM solution, with a model using 46 quadratic elements. Because the height of point $P$ is dependent upon the fillet radius, the effect of changing the run-out radius was found using a combination of two re-analysis operations—a move points operation followed by a resize fillet operation. For each move points operation, the point $P$ is dragged vertically, and the two lines joining at $P$ are remeshed. For each resize fillet operation, the fillet arc and both tangent lines are modified. The total number of elements on these lines is redistributed on the new line geometry, and the re-analysis proceeds.

For each move points operation, 30% of the elements are modified, a proportion that reduces to 24% for each resize fillet operation. The combined run time of the two re-analyses is less than that which would be required to analyse the single geometry again. However, the significance of the re-analysis approach in this case is not the overall run time saving, which is almost imperceptible for such a small model. Instead the benefits here come from the automatic update of stress contours with each geometric modification. On a Pentium III (500 MHz) processor, the re-analysis is almost instantaneous, taking 0.11 s. This lends a real-time stress analysis tool to the engineer, who can assess immediately the impact of a proposed design change on durability.

In this case, the reduction of maximum principal stress in the fillet is presented in Fig. 6.

This entire sequence of results may be achieved within a few seconds after constructing the first model. The design may be readily optimised. Providing an experienced engineer with a rapid re-analysis tool can be seen to guide a design optimisation procedure to a good engineering solution in a short space of time.

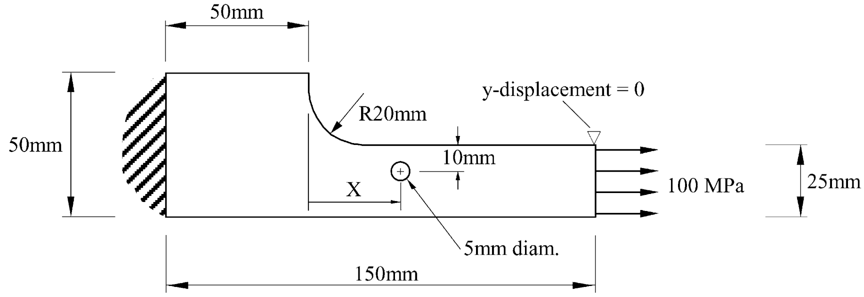


Fig. 7. Interacting stress concentrations problem.

A second problem that illustrates these ideas is shown in Fig. 7. This involves the interaction between two stress concentration effects, a circular hole and a fillet. The location of the hole is to be determined, as in this case it is desirable to position the hole as near as possible to the fillet, though without violating the design stress constraints. A series of tests considers a range of values of $X$ (as defined in the figure) from 0 to 60 mm.

Each geometric change is performed using a Move Shape operation, simply moving the circle to its new location. The interaction between the two stress concentrations, as the hole passes the fillet, is clearly shown in Fig. 8.

While of course this analysis can be performed by any software, finite element or boundary element, that allows parametric variation in the analysis, the freedom offered by the re-analysis ideas presented here allow decisions to be made more rapidly on the information gained through the automatically revised models. For the purposes of this illustration, Fig. 8 shows the variation in $K_T$ as the dimension $X$ increases. However, it is important to recognise that the design modifications need not take place monotonically, so that $X$ is always increasing. Indeed, non-montonic variation of parametric dimensions is a better exploitation of the interactive analysis offered by the scheme described, for as soon as a change in a design variable is found to be detrimental, the engineer can act to change the geometry back towards the earlier (better) design.

It is valid to ask the question 'is this optimisation?' Clearly, it is not optimisation in the sense of mathematical techniques such as non-linear programming, or any of the numerous analytical approaches for shape and topology optimisation. However, this does offer a rapid method of approaching an 'engineering optimum', in which the engineer is using his/her experience to guide the design to a good, robust solution in the design space. In addition, any required ease of manufacturability is ensured by the engineer, who may dictate that straight edges remain straight, etc. Trial and error is a very powerful method in the hands of a competent engineer with the right tools. For engineers looking to use mathematical approaches to optimisation, re-analysis is still valuable since the availability of rapid solutions to perturbed geometries is highly desirable in this context also.

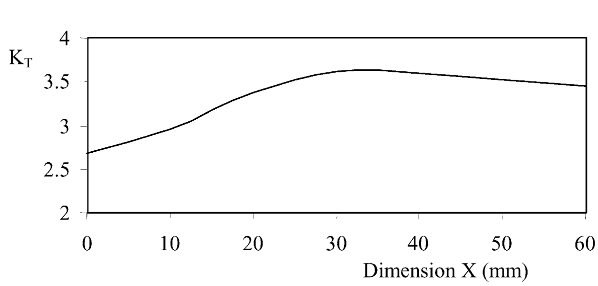


Fig. 8. Stress concentration factor, $K_T$, for various hole locations.

## 6. Conclusions

A simple methodology has been presented for the use of the BEM in mechanical design. The essential elements are:

- The overlaying of an automated BEM solution onto a simple sketching tool,
- The automatic re-analysis of a model following a geometric design change.

The re-analysis method has the advantage of its simplicity. The model is remeshed following the design change, and the recalculation of the BEM system matrix limited to only those parts that require changing since the previous design. An iterative solver computes the new solution vector, using the previous solution vector as a suitable first guess. This approach is competitive with previously published re-analysis schemes in terms of computational performance, and has the distinct advantage that the system matrix is continually updated to relate to the latest design geometry. This allows the stress solutions to evolve accurately and efficiently with the design geometry.

A 3D implementation of these ideas has been added to the 2D software. The 3D re-analysis offers performance improvements of similar order to the 2D case, in comparison with the full analysis, and its dependence on the percentage of elements changed follows a similar trend. Some differences in the character of the re-analysis have been noted, including the better reward in 3D for the good first guess to the solution vector than is found in 2D resolution.

Element distortion during re-meshing has been found to have a detrimental effect on re-analysis performance. The optimisation of the re-meshing is shown to be a key element in the success of a 3D re-analysis scheme.

Examples of the usage of software based on these ideas in an industrial setting have been presented to illustrate the concepts.

It is noted that the availability of a rapid solution to a slightly modified geometry is of great benefit to a variety of mathematical shape optimisation schemes. However, providing an experienced engineer with a rapid and accurate tool to sketch, analyse and re-analyse a component is seen as a very powerful method of design improvement, leading to an engineering optimum.

## Acknowledgements

The work described in this paper was carried out under a research grant GR/L73128 from the Engineering and Physical Sciences Research Council (EPSRC), UK. The authors also thank Damian Scott (BAE SYSTEMS) for development of analysis models.

## References

[1] Trevelyan J. Boundary elements for engineers: theory and application. Southampton: Computational Mechanics Publications, 1994.

[2] Telles JCF. A self-adaptive coordinate transformation for efficient numerical evaluation of general boundary element integrals. Int J Numer Meth Engng 1987;24:959–73.

[3] Saad Y, Schultz MH. GMRES: a generalized minimum residual algorithm for solving nonsymmetric linear systems. SIAM J Sci Stat Comput 1986;7(3):856–69.

[4] Arora JS. Survey of structural reanalysis techniques. J Struct Div, ASCE 1976;102(ST4):783–802.

[5] Abu Kassim AM, Topping BHV. Static reanalysis: a review. J Struct Engng, ASCE 1987;113(5):1029–45.

[6] Mackie RI. An object-oriented approach to fully interactive finite element software. Adv Engng Software 1998;29(2):139–49.

[7] Press WH, Flannery BP, Teukolsky SA, Vetterling WT. Numerical recipes. Cambridge: Cambridge University Press, 1986.

[8] Castillo E, Cobo A, Fernández-Canteli A, Jubete F, Pruneda E. Updating inverses in matrix analysis of structures. Int J Numer Meth Engng 1998;43:1479–504.

[9] Kirsch U, Toledano G. Approximate re-analysis for modifications of structural geometry. Comput Struct 1983;16(1):269–79.

[10] Kane JH, Keshava Kumar BL, Gallagher RH. Boundary element iterative reanalysis for continuum structures. J Enng Mech, ASCE 1990;116(10):2293–309.

[11] Leu L-J. Shape optimization by the boundary element method with a reduced basis reanalysis technique. Struct Engng Mech 1999;8(1): 73–84.

[12] Noor AK. Recent advances and applications of reduction methods. Appl Mech Rev 1994;17:125–46.

[13] Kirsch U, Moses F. An improved re-analysis method for grillage type structures. Comput Struct 1998;68(1–3):79–88.

[14] Trevelyan J, Wang P. Interactive re-analysis in mechanical design evolution. Part I: background and implementation. Comput Struct 2001;79(9):929–38.

[15] Trevelyan J, Wang P. Interactive re-analysis in mechanical design evolution. Part II: rapid evaluation of boundary element integrals. Comput Struct 2001;79(9):939–51.

[16] Peterson RE. Stress concentration factors. New York: Wiley, 1974.